MONTHLY WEATHER REVIEW
VOLUME 96, NUMBER 4
APRIL 1968

A DIAGNOSTIC BALANCE MODEL FOR STUDIES OF WEATHER SYSTEMS OF LOW AND HIGH LATITUDES, ROSSBY NUMBER LESS THAN 1

T. N. KRISHNAMURTI

Department of Meteorology, Florida State University, Tallahassee, Fla.

ABSTRACT

This paper outlines a theory for a diagnostic balance model. A unique manner of partitioning baroclinic vertical motions into various forcing mechanisms is proposed as a natural extension of the quasi-geostrophic problem. Forcing functions include advection of vorticity and temperature by the nondivergent and the divergent part of the wind. Role of various terms of the complete vorticity and the so-called balance equations are included in the analysis. Other features of the diagnostic model are air flow over terrain, frictional contributions at the lower boundary, sensible heat transfer from water surfaces, and stable and unstable formulations of latent heat release. Typical magnitudes and physical interpretations of several nongeostrophic mechanisms are illustrated. Two applications of the above mentioned diagnostic model appear in this issue, a study of a frontal cyclone development by Krishnamurti and a study of a low latitude disturbance by Baumhefner.

1. INTRODUCTION

A diagnostic balance model can be used to describe the three dimensional motion, temperature, pressure, and moisture fields consistent with a system of scaled dynamical equations. The diagnostic balance model provides a detailed initial state for a primitive equation model and may be used for studies of short period evolution of weather systems. Adjustment of wind and pressure field in a primitive equation model may be studied from such an input and from simpler balanced systems.

In the following we present dynamical equations that are valid for a small Rossby number ($R_o<1$) theory; Lorenz [10] and Phillips [13] have discussed the essential scale analysis and energetics of this system of equations. In the middle latitudes, cyclone scale disturbances belong to this class, namely

$$R_o=U/fL<1;$$

however the possible application of a dynamical system where $R_o>1$ in low latitudes is questionable.

The model can be applied with considerable confidence to study weather systems of the Tropics that are found about 5° lat. away from the Equator and that have not reached hurricane strength. There are a large number of tropical disturbances outside of the intertropical convergence zone and the hurricane class where the Rossby number is still <1 and the expansion theory is hence valid.

In a typical easterly wave, for instance,

$$u\approx 10 \text{ m.p.s.}$$
$$f\approx 0.5\times 10^{-4} \text{ sec.}^{-1}$$
$$L\approx 10^{6} \text{ m.}$$
$$R_o\approx 0.2<1.$$

If we deal with disturbances closer to the Equator or of wavelengths smaller than 1000 km. or much larger wind speeds, then this analysis would not apply. The types of weather disturbances we deal with here satisfy the criteria of small Rossby number. In this issue of the *Monthly Weather Review* we present a detailed study of an extratropical storm (Krishnamurti [7]) and a study of a nondeveloping easterly wave under an upper cold Low (Baumhefner [2]). Our studies also include an investigation of a developing easterly wave (Krishnamurti [6]). The calculations in this latter were carried out during its prehurricane stage when the flow fields were characterized by $R_o<1$. Other related studies appear in Krishnamurti [8].

An examination of large-scale motion, temperature, pressure, and moisture distribution is needed for any search of instability mechanism that may be important in the subsequent developments to large Rossby number phenomena.

The diagnostic balance model is a very powerful tool for studying the role of a number of rather complex mechanisms that describe the instantaneous state of the atmosphere. The formulation of the model for the middle latitudes is somewhat different from that in the Tropics for at least two essential reasons:

i) The nondivergent stream function of the horizontal velocity field for middle latitude systems can in general be obtained from a solution of a balance equation for a prescribed distribution of the geopotential height field. An analysis of the geopotential height field is not easily possible in the Tropics and a wind field is required to obtain a stream function which corresponds to the observed relative vorticity distribution.

ii) Heating functions describing effects of latent heat release can be handled rather easily if dynamical ascent of absolutely stable air is producing condensation. This is generally true of the large-scale precipitation from stable nonconvective cloud systems. The heating function can in such cases be defined nonzero if

1) Atmosphere is absolutely stable
2) Relative humidity is near 100 percent
3) Air is rising on a large scale.

These conditions are generally met in the middle latitudes. In the Tropics heating functions have to be defined for convective type of cloud forms. A formal parameterization of the cumulus-scale heating should perhaps be carried out in somewhat the same way as it is done for studies of tropical storms, e.g. Kuo [9]. In the Tropics a heating function may be defined if

1) Atmosphere is conditionally unstable
2) Net moisture convergence in vertical columns >0.

While we have separated the middle latitudes and the Tropics in two broad categories for defining heating function, in the real atmosphere there is a large overlap generally, and this must be borne in mind in treating weather systems. For instance, near the fronts of a middle latititude cyclonic disturbance all of the tropical conditions will generally be satisfied and a heating function in such regions should be accordingly defined. The problem thus becomes somewhat too complicated. For the present, we have made this broad division between the high and low latitude disturbances.

2. THE BALANCE EQUATIONS

The quasi-static equation of motion with pressure as a vertical coordinate may be written in the form (a list of symbols appears in table 1),

$$\frac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\cdot\nabla)\mathbf{V}+\omega\frac{\partial \mathbf{V}}{\partial p}-f\mathbf{V}\times\mathbf{k}=-g\nabla Z+\mathbf{F} \tag{1}$$

$$\frac{R}{p}\,\theta\left(\frac{p}{p_0}\right)R/c_p=-g\,\frac{\partial z}{\partial p}. \tag{2}$$

The continuity equation may be written in the form

$$\nabla\cdot\mathbf{V}=-\frac{\partial\omega}{\partial p}. \tag{3}$$

The first law of thermodynamics is expressed by the relation,

$$c_p\,\frac{T}{\theta}\left[\frac{\partial\theta}{\partial t}+\mathbf{V}\cdot\nabla\theta+\omega\,\frac{\partial\theta}{\partial p}\right]=H \tag{4}$$

where H is the diabatic heating per unit mass of air.

In the formulation of a diagnostic balance model, vorticity and divergence equations are derived from equations (1) and (2). From considerations of scale analysis, certain terms involving the time derivatives of divergence and advection by the divergent part of the wind are dropped generally (Lorenz [10], Phillips [13]).

By defining,

$$\mathbf{V}=\mathbf{k}\times\nabla\psi-\nabla\chi,$$

where ψ and χ define a stream function and velocity potential, we may write the vorticity and the divergence equations by the relations:

$$\frac{\partial}{\partial t}\nabla^2\psi=-J(\psi,\zeta_a)+\nabla\chi\cdot\nabla\zeta_a+\zeta_a\nabla^2\chi-\omega\,\frac{\partial}{\partial p}\nabla^2\psi$$
$$-\nabla\omega\cdot\nabla\,\frac{\partial\psi}{\partial p}-g\,\frac{\partial}{\partial p}\left[\frac{\partial\tau_y}{\partial x}-\frac{\partial\tau_x}{\partial y}\right] \tag{5}$$

$$\nabla\cdot f\nabla\psi=\nabla^2\phi-2J\left(\frac{\partial\psi}{\partial x},\frac{\partial\psi}{\partial y}\right). \tag{6}$$

The frictional force is retained at the 1000-mb. surface, and is defined in terms of stresses τ_x and τ_y.

$$\tau_x=C_D\rho U\sqrt{U^2+V^2} \tag{7}$$

$$\tau_y=C_D\rho V\sqrt{U^2+V^2} \tag{8}$$

where U and V are the total horizontal wind components on a pressure surface and C_D is a drag coefficient. The nondivergent stream function in the following is obtained from equation (6) for a given geopotential ϕ, for the high latitude weather systems. The method of solution is the same as that given in Shuman [14], except when the equation is hyperbolic over part of the area of interest, in which case we have solved the equation

$$\nabla\cdot f\nabla\psi=\nabla^2\phi-2J(U_g,V_g). \tag{9}$$

Comparison of the two stream functions obtained from equations (6) and (9) for an example with no hyperbolic regions showed very slight differences in the stream functions. Equation (9) may hence be used to evaluate the stream function when there are limited hyperbolic

TABLE 1.—*List of symbols, units, and typical magnitudes. Note that we use the following fundamental units: millibar$^{\alpha}$, meter$^{\beta}$, second$^{\gamma}$, degree$^{\delta}$. The table contains the symbol; meaning of symbol; values of α, β, γ, δ; and a typical atmospheric magnitude of the quantity in those units. The * denotes very wide range of magnitudes.*

Symbol	Meaning of symbol	α	β	γ	δ	Magnitude
a	Fractional area covered by convective clouds	0	0	0	0	10^{-2}
f	Coriolis parameter	0	0	−1	0	10^{-4}
u	Zonal velocity of total balance wind	0	1	−1	0	10
v	Meridional velocity of total balance wind	0	1	−1	0	10
ω	Vertical velocity	1	0	−1	0	10^{-3}
ρ	Density of air	1	−2	2	0	10^{-2}
x	Distance along zonal direction	0	1	0	0	10^{3}
y	Distance along meridional direction	0	1	0	0	10^{3}
p	Pressure, vertical coordinate distance	1	0	0	0	10^{2}
f_0	Mean value of Coriolis parameter	0	0	−1	0	10^{-4}
σ	Dry static stability	−2	2	−2	0	10^{-2}
σ_s	Moist static stability	−2	2	−2	0	*10^{-3}
R	Gas constant	0	2	−2	−1	2000/7
c_p	Specific heat of air of constant pressure	0	2	−2	−1	10^{3}
c_v	Specific heat of air at constant volume	0	2	−2	−1	5000/7
p_0	Reference pressure	1	0	0	0	1000
g	Acceleration of gravity	0	1	−1	0	10
$\mathbf{k}$	Unit vector in the vertical direction	0	1	0	0	1
T	Air temperature	0	0	0	1	300
θ	Potential temperature	0	0	0	1	300
$\mathbf{V}$	Horizontal velocity vector	0	1	−1	0	10
$\mathbf{V}_{\psi}$	Rotational part of horizontal wind vector	0	1	−1	0	10
$\mathbf{V}_{\chi}$	Divergent part of horizontal wind vector	0	1	−1	0	1
β	Northward variation of Coriolis parameter	0	−1	−1	0	10^{-11}
z	Height of constant pressure surfaces	0	1	0	0	*6000
ϕ	Geopotential of constant pressure surfaces	0	2	−2	0	*6×10^{4}
ψ	Rotational part of the horizontal wind	0	2	−1	0	10^{8}
χ	Velocity potential	0	2	−1	0	10^{6}
∇	Del operator	0	−1	0	0	10^{-3}
∇^2	Laplacian operator	0	−2	0	0	10^{-7}
J	Jacobian operator	0	−2	0	0	10^{-7}
π	Symbol for $RT/p\theta$	−1	2	−2	−1	*0.3
$F_i, A(\psi), B(\psi)$	Forcing functions of ω equation	−1	0	−3	0	*10^{-16}
q	Specific humidity	0	0	0	0	10^{-3}
q_s	Saturation specific humidity	0	0	0	0	10^{-3}
ζ_a	Absolute vorticity	0	0	−1	0	10^{-4}
Δx	Grid distance along x-axis	0	1	0	0	2.5×10^{5}
Δy	Grid distance along y-axis	0	1	0	0	2.5×10^{5}
F	Frictional force per unit mass	0	1	−2	0	*10^{-4}
ζ	Relative vorticity	0	0	−1	0	10^{-4}
L	Latent heat	0	2	−2	0	10^{6}
D_1, D_2	Components of deformation	0	0	−1	0	10^{-4}
H	Heating function	0	2	−3	0	10^{-2}
F_S	Sensible heat flux	1	1	−1	0	10^{-1}
I	Net moisture convergence	1	−1	1	0	10^{-6}
τ_x, τ_y	Frictional stress	1	0	0	0	10^{-3}
ω_t	Terrain induced vertical motion	1	0	−1	0	10^{-3}
M	Map scale factor	0	0	0	0	1.0
$\bar{\phi}$	Latitude	0	0	0	0	1.0

regions. The ellipticity condition is frequently expressed by the relation

$$\nabla^2\phi+\tfrac{1}{2}f^2-\nabla f\cdot\nabla\psi>0. \tag{10}$$

Since $\nabla\cdot(f\nabla\psi)\approx\nabla^2\phi$, the inequality expressed by relation (10) may be written as: $\nabla^2\psi>-f/2$ which is sometimes used as an approximate condition for ellipticity of the balance equation. This relation is generally satisfied by the solution over most regions except where the magnitude of the anticyclonic relative vorticity is large. For the low latitude system relative vorticity $\nabla^2\psi$ is obtained from the analyzed wind field (isotach and isogon distribution)

$$\nabla^2\psi=\frac{\partial V^o}{\partial x}-\frac{\partial U^o}{\partial y} \tag{11}$$

where U^o and V^o are the zonal and meridional components of the observed wind. This stream function is then assumed to be related to a geopotential height distribution from the balance equation,

$$\nabla^2\phi=\nabla\cdot f\nabla\psi+2J\left(\frac{\partial\psi}{\partial x},\frac{\partial\psi}{\partial y}\right). \tag{12}$$

There are several numerical problems that are encountered in the actual solution of these boundary value problems; we shall discuss these in some detail in the other sections.

The thermodynamic energy equation (4) may be combined with an equation of state,

$$\pi=\frac{RT}{p\theta} \tag{13}$$

and a relation for a static stability parameter,

$$\sigma=-\frac{RT}{p\theta}\frac{\partial\theta}{\partial p}=-\pi\frac{\partial\theta}{\partial p} \tag{14}$$

to obtain the following equation,

$$\pi\frac{\partial\theta}{\partial t}=-\pi J(\psi,\theta)+\pi\nabla\chi\cdot\nabla\theta+\sigma\omega+\frac{HR}{c_p p}. \tag{15}$$

The ω-equation of a general balance model is obtained by combining equations (5), (6), and (15). It is expressed by the following three equations for ω, χ, and $\frac{\partial\psi}{\partial t}$:

$$\begin{aligned}\nabla^2\sigma\omega+f^2\frac{\partial^2\omega}{\partial p^2}&=f\frac{\partial}{\partial p}J(\psi,\zeta_a)+\pi\nabla^2J(\psi,\theta)\\&-2\frac{\partial}{\partial t}\frac{\partial}{\partial p}J\left(\frac{\partial\psi}{\partial x},\frac{\partial\psi}{\partial y}\right)-f\frac{\partial}{\partial p}(\zeta\nabla^2\chi)\\&+f\frac{\partial}{\partial p}g\frac{\partial}{\partial p}\left[\frac{\partial\tau_y}{\partial x}-\frac{\partial\tau_x}{\partial y}\right]-\frac{R}{c_p p}\nabla^2H_L\\&-\frac{R}{c_p p}\nabla^2H_S+f\frac{\partial}{\partial p}\left(\omega\frac{\partial}{\partial p}\nabla^2\psi\right)\\&+f\frac{\partial}{\partial p}\left(\nabla\omega\cdot\nabla\frac{\partial\psi}{\partial p}\right)-f\frac{\partial}{\partial p}\left(\nabla\chi\cdot\nabla\zeta_a\right)\\&-\pi\nabla^2\left(\nabla\chi\cdot\nabla\theta\right)-\beta\frac{\partial}{\partial p}\frac{\partial}{\partial y}\frac{\partial\psi}{\partial t},\end{aligned} \tag{16}$$

$$\nabla^2\chi=\frac{\partial\omega}{\partial p}, \tag{17}$$

$$\begin{aligned}\nabla^2\frac{\partial\psi}{\partial t}&=-J(\psi,\zeta_a)-g\frac{\partial}{\partial p}\left[\frac{\partial\tau_y}{\partial x}-\frac{\partial\tau_x}{\partial y}\right]\\&+\nabla\chi\cdot\nabla\zeta_a+\zeta_a\nabla^2\chi\\&-\nabla\omega\cdot\nabla\frac{\partial\psi}{\partial p}-\omega\frac{\partial}{\partial p}\nabla^2\psi.\end{aligned} \tag{18}$$

Equations (16), (17), and (18) are solved by numerical techniques, and a discussion of these solutions is a major part of this paper. There are several problems that are encountered in the solution of the equations that are of interest:

i) Proper boundary conditions for ω, χ, and $\frac{\partial\psi}{\partial t}$

ii) Ellipticity of the three equations, validity of boundary value techniques

iii) Finite difference analogs of the three equations

iv) Formulation of heating terms, sensible and latent heat for stable and unstable situations

v) Formulation of surface friction

vi) Inclusion of terrain effects

vii) Interpretation of results.

3. THE DEVELOPMENT PROBLEM

For a simple quasi-geostrophic theory (adiabatic, frictionless flow) the individual change of vorticity is given by the convergence of mass

$$\left(\frac{\partial}{\partial t}+\mathbf{V}\cdot\nabla\right)\zeta_a=f_0\frac{\partial\omega}{\partial p}. \tag{19}$$

Vertical motion in a quasi-geostrophic theory is given by the ω-equation,

$$\nabla^2\sigma_{(p)}\omega+f_0^2\frac{\partial^2\omega}{\partial p^2}=f_0\frac{\partial}{\partial p}\mathbf{V}\cdot\nabla\zeta_a-\nabla^2\mathbf{V}\cdot\nabla\frac{\partial\phi}{\partial p}. \tag{20}$$

The two forcing functions of an ω-equation are: i) Differential vorticity advection, and ii) Laplacian of thermal advection.

Given the ϕ distribution for several map times the instantaneous distribution of vertical motions can be determined from a solution of the ω-equation. If three dimensional trajectories are constructed utilizing these velocities, then the observed change in vorticity along the trajectory may be related to the computed convergence and we may hence determine the map features that contribute to vertical motions and convergence. Sutcliffe's [15] and Petterssen's [12] development criteria are essentially quasi-geostrophic development formulas that pursue this sort of reasoning.

In a general balance model this problem becomes somewhat more complicated. With the ψ and χ distribution of a balance model having been obtained, the horizontal velocity components

$$U=-\frac{\partial\chi}{\partial x}-\frac{\partial\psi}{\partial y} \tag{21}$$

$$V=-\frac{\partial\chi}{\partial y}+\frac{\partial\psi}{\partial x} \tag{22}$$

and the p-components for several map times may be used to construct three dimensional trajectories (Paegle [11]). Along such trajectories various development terms of the vorticity equation may be tabulated and a listing of various baroclinic mechanisms producing rising motions and convergence can be made. All of this information becomes very useful for studying the storm development problem. This has been a major motivation for pursuing this investigation.

4. FORCING FUNCTIONS OF THE BALANCE ω-EQUATION

The quasi-geostrophic ω-equation contains two forcing functions, while the general balance ω-equation, presented here, has 12 forcing functions. At first sight it is not quite obvious that an analysis should be carried this far, but as we shall show the contribution by several of these terms is quite large and yields information that is not obtainable from a quasi-geostrophic model.

The forcing functions are:

1. $f\frac{\partial}{\partial p}J(\psi,\zeta_a)$	Differential vorticity advection by the nondivergent part of the wind
2. $\pi\nabla^2 J(\psi,\theta)$	Laplacian of thermal advection by the nondivergent part of the wind
3. $-2\frac{\partial}{\partial t}\frac{\partial}{\partial p}J\left(\frac{\partial\psi}{\partial x},\frac{\partial\psi}{\partial y}\right)$	We label this term as a differential deformation effect (explained later).
4. $-f\frac{\partial}{\partial p}(\zeta\nabla^2\chi)$	Differential divergence effects of a balance model
5. $f\frac{\partial}{\partial p}g\frac{\partial}{\partial p}\left[\frac{\partial\tau_y}{\partial x}-\frac{\partial\tau_x}{\partial y}\right]$	Effects of frictional stresses
6. $-\frac{R}{c_p p}\nabla^2 H_L$	Effects of latent heat
7. $-\frac{R}{c_p p}\nabla^2 H_S$	Effects of sensible heat transfer from water surfaces to the atmosphere
8. $f\frac{\partial}{\partial p}\left(\omega\frac{\partial}{\partial p}\nabla^2\psi\right)$	Differential vertical advection of vorticity
9. $f\frac{\partial}{\partial p}\left(\nabla\omega\cdot\nabla\frac{\partial\psi}{\partial p}\right)$	Differential turning of vortex tubes
10. $-f\frac{\partial}{\partial p}\{\nabla\chi\cdot\nabla\zeta_a\}$	Differential advection of vorticity by the divergent part of the wind
11. $-\pi\nabla^2\{\nabla\chi\cdot\nabla\theta\}$	Laplacian of thermal advection by the divergent part of the wind
12. $-\beta\frac{\partial}{\partial p}\frac{\partial}{\partial y}\frac{\partial\psi}{\partial t}$	Contribution by the beta term of the divergence equation.

The complete problem is solved with and without terrain effects to estimate terrain contribution. It might have been desirable to include the terrain effects as an internal forcing function. The pressure frame (x, y, p) is somewhat artificial near the lower boundary due to reduction of data to sea level, and this boundary condition at best is only a compromise for the real problem of upslope and downslope motion of air. An earth frame, where the earth's surface is a coordinate surface, would be more desirable for inclusion of terrain boundary condition, but the problem becomes many times more complicated when all the terms of the balance model are retained.

The problem, as we have posed it, contains 12 internal forcing functions and one external forcing function.

In all cases the following qualitative rule is found important for interpreting vertical motion contribution. If a forcing function F_i is greater than zero, then in the vicinity of this region its contribution will be rising motions. There are exceptions to this rule but generally this is true.

It is thus easy to verify the relation between rising and sinking motion and the vorticity and thermal advection patterns. Synoptic experience and results of simple quasi-geostrophic ω-equation solutions have verified the inverse relationship between these forcing functions and the sign of rising or sinking motion. It must, however, be noted that the two terms of the quasi-geostrophic theory do not exactly correspond to the first two terms of a balance model. The stream function for the former case is the geostrophic stream function gz/f_0 while for the later case it is the balance nondivergent stream function. Besides, in the quasi-geostrophic theory f is replaced by f_0 and the static stability σ is a function of pressure only, while in the balance model the contribution from the first two terms arise for variable f and σ (x, y, p).

A qualitative interpretation of terms 1, 2, 8, 9, 10, and 11 can be made in a similar manner. The heating terms 6 and 7 will be positive if H_L and H_S are positive and will in general contribute rising motions. The forms of these functions H_L and H_S are discussed in a separate section. Frictional stresses (term 5) will generally contribute rising motions in regions of cyclonic relative vorticity and sinking motions in regions of anticyclonic relative vorticity.

The deformation and the divergence terms (3 and 4) do not appear in the quasi-geostrophic theory but are large (as we shall demonstrate later) and their interpretation is somewhat difficult.

Let

$$D_1=\frac{\partial U_\psi}{\partial x}-\frac{\partial V_\psi}{\partial y} \tag{23}$$

and

$$D_2=\frac{\partial V_\psi}{\partial x}+\frac{\partial U_\psi}{\partial y} \tag{24}$$

define the two components of the deformation field. Then we can show that

$$D_1^2+D_2^2-\zeta^2=-4J(U_\psi,V_\psi). \tag{25}$$

In the vicinity of intensifying frontal zones the magnitude of deformation generally increases and the contribution by the term

$$-2\frac{\partial}{\partial t}\frac{\partial}{\partial p}J(U_\psi,V_\psi)$$

will be expected to be large. Much further analysis of this term is needed.

The divergence term strongly modifies the vertical motion distribution produced by the leading two terms in regions where ζ is large. This is easy to see from the form of the forcing function,

$$-f\frac{\partial}{\partial p}\zeta\nabla^2\chi.$$

Let $\zeta>0$ in a region of strong sinking motion, at low levels $\zeta\nabla^2\chi<0$, and at upper levels $\zeta\nabla^2\chi>0$; hence

$$-f\frac{\partial}{\partial p}\zeta\nabla^2\chi>0$$

and will contribute rising motion and oppose the two leading terms. The converse holds in regions of strong rising motions.

The β term (term 12) is the least important of all the terms listed above.

We have not included any radiative effects in the analysis presented here; the assumption has been that measures of the instantaneous tendencies of atmospheric variables for synoptic scale motions can be made without invoking these effects. This may prove to be wrong. We shall next discuss some of the more detailed aspects of heating, friction, and terrain effects in the model.

5. THE HEATING FUNCTION OF THE ω-EQUATION

In the ω-equation the forcing function of heating terms is written in the form,

$$F_H=-\frac{R}{c_p p}\nabla^2 H \tag{26}$$

where H is defined through the first law of thermodynamics as the rate of heating per unit mass of air

$$c_p\frac{T}{\theta}\frac{d\theta}{dt}=H. \tag{27}$$

In our analysis we have restricted H to contain the effects of sensible heat transfer from the water surfaces H_S, and the effects of release of latent heat H_L in the free atmosphere, thus $H=H_S+H_L$.

SENSIBLE HEAT

Transfer of sensible heat from ocean surface is very important in examples of strong polar outbreaks of cold air over warm oceans. We have included a heat flux equation defined by the empirical relation,

$$F_S=C(T_w-T_a)v. \tag{28}$$

Jacobs [5] used a value $C=4.7\times10^{-3}$, when the units of the various quantities are

$$F_S\text{: cal./cm}^2\text{./min.}$$

$$v\text{: m./sec.}$$

$$T_w-T_a\text{: °C.}$$

The rate of heating H_S is measured as a convergence of flux, i.e.

$$H_S=g\frac{\partial F_S}{\partial p}. \tag{29}$$

We have assumed $F_S=0$ above the 900-mb. surface in our studies; thus the forcing function for the ω-equation at the 900-mb. surface has the form,

$$F_{HS}=-\frac{R}{c_p p}g\nabla^2\frac{F_S(1000)}{\Delta p}$$

$$=-\frac{Rg}{c_p p\,\Delta p}\nabla^2 F_S(1000). \tag{30}$$

The forcing function has dimension sec.$^{-3}$ mb.$^{-1}$; thus F_S should have the dimension mb. m. sec.$^{-1}$ Hence we may write,

$$F_S=32.9\times10^{-3}\,V(T_w-T_a)\text{ mb. m. sec.}^{-1} \tag{31}$$

and

$$F_{HS}=4.5953\times10^{-7}\nabla^2V(T_w-T_a)\text{ in units of sec.}^{-3}\text{ mb.}^{-1} \tag{32}$$

A crude measure of the vertical velocity near the 1000-mb. surface can be made by writing,

$$\omega_S=-\frac{4.5953\times10^{-7}}{\sigma}V(T_w-T_a)\text{ in units of mb./sec.} \tag{33}$$

If we assume typical values of the wind speed ($V=10$ m.p.s.), sea-air temp. difference ($T_w-T_a=1$°C.) and static stability ($\sigma=0.02$ m.^{2} sec.$^{-2}$ mb.$^{-2}$) then we obtain,

$$\omega_S\approx-2.3\times10^{-4}\text{ mb./sec.}$$

which is a rising motion of about 0.3 cm./sec. The corresponding $\partial\omega_s/\partial p$ and rate of production of vorticity at the sea level will thus be a small quantity. In strong polar outbreaks V can be as large as 25 m.p.s. and (T_w-T_a) as large as 10°C., and σ may be quite variable, corresponding contribution to vertical motions near the 1000-mb. surface may well exceed 1 cm./sec. In our studies the forcing function for the sensible heat is given by the expression,

$$F_{HS}=-4.5953\times10^{-7}\,\nabla^2V(T_w-T_a). \tag{34}$$

In the nonlinear balance model static stability is permitted to vary in the x, y, p space; thus a realistic measure of the effect of sensible heat is possible. It must however be noted that the empirical coefficients of the Jacobs transfer formula are not very reliable and such calculations need considerable refinement.

LATENT HEAT

The following static stability parameters are relevant to our studies.

$$\sigma=-\frac{RT}{p\theta}\frac{\partial\theta}{\partial p}\text{ (dry)} \tag{35}$$

and

$$\sigma_e=-\frac{RT}{p\theta_e}\frac{\partial\theta_e}{\partial p}\text{ (moist)} \tag{36}$$

where θ_e is the equivalent potential temperature defined by the relation,

$$\theta_e=\theta\exp(Lq_s/c_pT). \tag{37}$$

The relation between σ_e and σ is given by the approximate relation,

$$\sigma_e=\sigma-\frac{RL}{c_p p}\frac{\partial q_S}{\partial p}. \tag{38}$$

Since $\partial q_S/\partial p$ is large in the lower latitudes and in the lower troposphere below 700 mb. generally, σ_e can be negative or positive depending on the magnitude of the second term in equation (38). According to parcel ascent considerations we define stability by the inequalities:

i) $\sigma>0$ Absolutely stable
$\sigma_e>0$
ii) $\sigma>0$ Conditionally unstable
$\sigma_e<0$
iii) $\sigma<0$ Absolutely unstable
$\sigma_e<0$.

The dry static stability is regarded as a function $\sigma(p)$ in quasi-geostrophic models. In the nonlinear balanced model static stability is permitted to vary in the three dimensions.

In our studies of synoptic scale motions we do not have to deal with absolutely unstable regions, but inequalities i) and ii) do appear. Middle latitude temperature distributions are characterized by relation i), and tropical flow below 700 mb. satisfies the relation ii) generally. In summer large areas of the lower troposphere as far north as 40° lat. can be conditionally unstable, and in wintertime during periods of strong polar outbreaks tropical latitudes may be absolutely stable on the synoptic scale.

For the absolutely stable case we define a heating function H_L by the relation,

$$H_L=-L\omega\frac{\partial q_S}{\partial p} \tag{39}$$

provided the air is nearly saturated, rising, and $\sigma_e>0$.

Since the effective static stability is positive, the ω-equation remains elliptic. For the conditionally unstable case we evaluate a heating function by parameterizing the subgrid scale heating as a function of the net convergence of moisture on the synoptic scale, essentially along the lines of Charney and Eliassen [3], or Kuo [9].

$$\text{Let } I=\frac{1}{g}\int_{p_B}^{p_T} \nabla\cdot q\mathbf{V}dp-\frac{\omega_B q_B}{g}. \tag{40}$$

I gives a measure of net convergence of moisture in vertical columns extending from the top of the friction layer (subscript B) to the top of the atmosphere (subscript T).

The corresponding heating function H_L may be written as,

$$H_L=Lg\,\frac{1}{q_{SB}}\frac{\partial q_S}{\partial p}\,A\,I \tag{41}$$

where A is an arbitrary coefficient, and measures the fraction of I that will go into the formation of convective elements. A direct calculation of this coefficient can be made in a manner shown by Kuo [9], as we show elsewhere (Krishnamurti [8]). The corresponding forcing function is given by the expression,

$$-\frac{R}{c_p p}\nabla^2 H_L=-\frac{RLg}{c_p p}\nabla^2\frac{1}{q_{SB}}\frac{\partial q_S}{\partial p}A\,I. \tag{42}$$

In Kuo's formulation H is expressed by the relation,

$$H=\frac{c_p}{\Delta t}\,a(T_S-T),$$

where T_S, Δt, and a are defined in table 1.

Hence we may write:

$$\text{Forcing function}=-\frac{R}{p\Delta t}\nabla^2\left[a(T_S-T)\right]. \tag{43}$$

A crude measure of large-scale vertical velocity arising from the parameterization of cumulus scale motion may be obtained by equating,

$$\nabla^2\sigma\omega\approx-\frac{R}{p\Delta t}\nabla^2 a(T_S-T)$$

or

$$\omega\approx-\frac{R}{p\Delta t\sigma}\,a(T_S-T).$$

Typical magnitudes are:

$$R\approx\frac{2000}{7}\text{ m.}^2\text{ sec.}^{-2}\text{ deg.}^{-1}$$
$$p\approx 500\text{ mb.}$$
$$\Delta t\approx 7200\text{ sec.}$$
$$\sigma\approx 0.02\text{ m.}^2\text{ sec.}^{-2}\text{ mb.}^{-2}$$
$$a\approx 0.05$$
$$T_S-T\approx 1°.$$

Hence

$$\omega\approx-2\times10^{-4}\text{ mb./sec.}$$

or

$$W\approx 0.25\text{ cm./sec. rising motion.}$$

The corresponding mid-tropospheric vertical velocity is very small. Formal solutions of ω-equations (without the approximations) do indeed yield vertical motion of this order or somewhat smaller in most of the tropical situations we have examined.

6. SURFACE FRICTION

The contribution from a frictional stress at the 1000-mb. surface, equations (7) and (8), is expressed by the following forcing function of the ω-equation:

$$F_F=-\frac{fC_D g}{10R}\left[\frac{\partial}{\partial y}\frac{U\sqrt{U^2+V^2}}{T}-\frac{\partial}{\partial x}\frac{V\sqrt{U^2+V^2}}{T}\right]. \tag{44}$$

The quantity in the brackets is evaluated from the divergent and the nondivergent component of the wind and temperature at the 1000-mb. surface. The manner by which the divergent part of the wind is evaluated in successive approximation procedure is discussed in another section. Since the choice of the value of the drag coefficient is very important in equation (44), it is somewhat unfortunate that a value based on earlier studies is the best that can be done at this stage. This is not very critical for middle latitude storms where, as we shall see, frictional vertical motion in cyclonic disturbances is around 1–2 cm./sec. at the lower levels and damps very rapidly with height. This vertical motion is generally overpowered by the vertical motions induced by the baroclinic dynamics and latent heat. This effect however becomes very important in the Tropics because low level wind speeds still are about the same order (10 m.p.s.), density of air is still about the same, and for a drag coefficient of 2.5×10^{-3} units, vertical velocities produced by frictional stresses are of the same order or larger than those produced by the weak baroclinic dynamics of the tropical weather systems.

7. TERRAIN (UPSLOPE AND DOWNSLOPE) VERTICAL MOTIONS

Terrain effect is introduced at the 1000-mb. surface as a lower boundary condition:

$$\omega_t=-g\,\frac{1000}{RT}\left[J(\psi,h)-\nabla\chi\cdot\nabla h\right] \tag{45}$$

where T, ψ, and χ are the values at the 1000-mb. surface and h is a smoothed terrain height obtained from a study of Berkofsky and Bertoni [1]. The ω_t was used as a lower boundary condition in our studies at the 1000-mb. surface during the relaxation of the ω-equation. This lower boundary effect varies each time a new value of χ is estimated, the second term is generally much smaller than the first, and a numerical scheme exhibits a rapid convergence. (A discussion of the manner in which ψ, ω, χ, and $\partial\psi/\partial t$ are evaluated is given below.)

8. THE CONCEPT OF PARTITIONING OF VARIOUS EFFECTS IN DIAGNOSTIC BALANCED MODELS

Consider the system of partial differential equations,

$$\nabla^2\sigma\omega+f^2\frac{\partial^2\omega}{\partial p^2}=A(\psi)+B(\psi)+L_1\left(\omega,\chi,\frac{\partial\psi}{\partial t},\psi\right)+L_2\left(\omega,\chi,\frac{\partial\psi}{\partial t},\psi\right)+\ldots+L_n\left(\omega,\chi,\frac{\partial\psi}{\partial t},\psi\right) \quad (46)$$

$$\nabla^2\chi=\frac{\partial\omega}{\partial p} \quad (47)$$

$$\nabla^2\frac{\partial\psi}{\partial t}=M_1(\omega,\chi,\psi)+M_2(\omega,\chi,\psi)+\ldots+M_n(\omega,\chi,\psi) \quad (48)$$

where $A(\psi)$, $B(\psi)$ are the leading forcing functions of the problem and $\psi(x, y, p)$ is prescribed. L_1, L_2, L_n, M_1, M_2, . . ., M_n are operators defining various terms of a general balance problem.

The corresponding quasi-geostrophic problem is defined by a single equation

$$\nabla^2\sigma(p)\omega+f_0^2\frac{\partial^2\omega}{\partial p^2}=A(\psi)+B(\psi). \quad (49)$$

For homogeneous boundary conditions,

$$\omega=0 \text{ at } x=x_1, x_2$$
$$y=y_1, y_2$$
$$\text{and } p=p_1, p_2,$$

we may write the quasi-geostrophic problem into the following two equations:

$$\nabla^2\sigma(p)\omega_A+f_0^2\frac{\partial^2\omega_A}{\partial p^2}=A(\psi) \quad (50)$$

and

$$\nabla^2\sigma(p)\omega_B+f_0^2\frac{\partial^2\omega_B}{\partial p^2}=B(\psi) \quad (51)$$

where

$$\omega_A+\omega_B=\omega. \quad (52)$$

A partitioning of vertical motion is thus possible for a quasi-geostrophic problem; ω_A is a contribution to the vertical motion from the forcing function $A(\psi)$ which may be the differential vorticity advection effects and ω_B the contribution by the thermal effects. Such a partitioning cannot in general be done using inhomogeneous boundary conditions (like terrain contributions) because they will enter into both ω_A and ω_B and the relation $\omega=\omega_A+\omega_B$ will not hold. A problem of inhomogeneous boundary condition can however be transformed into a problem with homogeneous boundary condition by redefining a dependent variable ω^* as a function of ω and the boundary effects. This is somewhat simple for the quasi-geostrophic case but becomes very complicated when we deal with the general balance model. Hence the partitioning that we shall discuss deals with the problem of homogeneous boundary conditions.

For the general balance problem there are at least two interesting ways of partitioning vertical motion distribution, which yield convergent solutions for ω, χ, and $\partial\psi/\partial t$.

PROBLEM 1

Let $L_1=L_2=\ldots=L_n=0$, solve

$$\nabla^2\sigma\omega^1+f^2\frac{\partial^2\omega^1}{\partial p^2}=A(\psi)+B(\psi), \quad (53)$$

and obtain ω^1, a first guess solution. This first guess should in general give the principal results, as is the case for the small Rossby number theory.

Next we write,

$$\nabla^2\chi^1=\frac{\partial\omega^1}{\partial p^1} \quad (54)$$

and

$$\nabla^2\left(\frac{\partial\psi}{\partial t}\right)^1=M_1(\omega^1,\chi^1,\psi)+\ldots+M_n(\omega^1,\chi^1,\psi). \quad (55)$$

Solutions for χ^1 and $(\partial\psi/\partial t)^1$ are then evaluated for the homogenous boundary conditions, and ω^2, χ^2, $(\partial\psi/\partial t)^2$, . . ., ω^n, χ^n, $(\partial\omega/\partial t)^n$ are then successively evaluated by retaining all the terms on the right side. Numerical convergence is defined by a set of small numbers ϵ_1, ϵ_2, and ϵ_n such that finite difference analogs of equations (46), (47), and (48) are satisfied to this degree of tolerance error. This is a numerical procedure that has been found to converge in a large number of examples where a distribution of $\psi(x, y, p)$ is assumed given.

The preceding discussion tacitly assumes that the equations for ω, χ, and $\partial\psi/\partial t$ are always elliptic. The ω-equation can become hyperbolic over portions of the region of interest for a variety of reasons. The principal problem in this regard, as mentioned earlier, arises from the heating functions. This problem can also arise in various other terms of the ω-equation, such as terms 4 and 9 of the master list of forcing functions. This is found to be true only when small mesh size of the order of 100 km. or less is considered. For larger scale flows it is adequate if the heating function and the effective static stability are properly retained.

With a convergent solution for ω, χ, and $\partial\psi/\partial t$ having been obtained, the partitioning of the vertical velocity may be carried out in the following manner:

$$\nabla^2\sigma\omega_A+f^2\frac{\partial^2\omega_A}{\partial p^2}=A(\psi) \quad (56)$$

$$\nabla^2\sigma\omega_B+f^2\frac{\partial^2\omega_B}{\partial p^2}=B(\psi) \quad (57)$$

$$\nabla^2\sigma\omega_n+f^2\frac{\partial^2\omega_n}{\partial p^2}=L_n\left(\omega,\chi,\frac{\partial\psi}{\partial t},\psi\right) \quad (58)$$

where ω_A and ω_B determine the contributions from the two principal forcing functions and ω_n are the contributions from n other terms. This partitioning has the feature that

$$\omega=\omega_A+\omega_B+\sum_{n=1}^{N}\omega_n$$

which is the total vertical velocity, and it yields considerable information regarding baroclinic processes that are part of the complex atmospheric phenomena. The operator on the left hand side contains the same terms as in the quasi-geostrophic theory; hence this manner of partitioning may be considered a natural extension of the quasi-geostrophic problem.

PROBLEM 2

Another mechanistic view of the partitioning problem is as follows. We write three equations in the form,

$$\nabla^2\sigma\omega+f^2\frac{\partial^2\omega}{\partial p^2}+\sum_n L_n\left(\omega, \chi, \frac{\partial\psi}{\partial t}, \psi\right)=A(\psi)+B(\psi) \quad (59)$$

$$\nabla^2\chi=\frac{\partial\omega}{\partial p} \quad (60)$$

$$\nabla^2\frac{\partial\psi}{\partial t}=\sum_n M_n(\omega, \chi, \psi), \quad (61)$$

In this formulation nonlinear terms like the twisting term, advections by the divergent part of the wind, etc. appear on the left hand side of the ω-equation. Their nature is similar to that of complex differential operators, except that not all of these operators contain ω explicitly. The pertinent question that one might ask with this formulation is what individual total contribution to ω arise from $A(\psi)$ and $B(\psi)$ respectively. There is no simple iterative scheme for solving this problem when the operator L_n appears on the left hand side, though the correct answer to the question raised here can be obtained by the following procedure.

Solve the system of equations

$$\nabla^2\sigma\omega+f^2\frac{\partial^2\omega}{\partial p^2}=A(\psi)+\sum_n L_n\left(\omega, \chi, \frac{\partial\psi}{\partial t}, \psi\right) \quad (62)$$

$$\nabla^2\chi=\frac{\partial\omega}{\partial p} \quad (63)$$

$$\nabla^2\frac{\partial\psi}{\partial t}=\sum_n M_n(\omega, \chi, \psi) \quad (64)$$

by dropping $B(\psi)$, by pursuing exactly the same procedure as in problem 1 outlined earlier. Let the final values of ω be $\omega(A_\psi)$.

Next a solution of the system of equations,

$$\nabla^2\sigma\omega+f^2\frac{\partial^2\omega}{\partial p^2}=B(\psi)+\sum_n L_n\left(\omega, \chi, \frac{\partial\psi}{\partial t}, \psi\right) \quad (65)$$

$$\nabla^2\chi=\frac{\partial\omega}{\partial p} \quad (66)$$

$$\nabla^2\frac{\partial\psi}{\partial t}=\sum_n M_n(\omega, \chi, \psi) \quad (67)$$

is obtained. Let the final values be $\omega=\omega(B_\psi)$. Then $\omega=\omega(A_\psi)+\omega(B_\psi)$ will give the unique total vertical velocity provided *there are no quadratic or higher order nonlinearities in* L_n *and* M_n; in our problem this would at least require a slight reformulation of the friction terms.

In simpler terminology we have in this second problem, modified the contribution to the rising motion by the differential vorticity advection and the thermal advection by building up the complex operators on the left hand side. This partitioning may in some ways be more realistic because now we have the same form of the two forcing functions as in the quasi-geostrophic problem on the right hand side. It is however somewhat hard to perceive physically why, for instance, frictional stresses or latent heat should modify instantaneous vertical motions produced by differential vorticity advection, and there are a number of such other questions that are hard to answer. I have preferred to determine the partitioning according to problem 1 discussed above because the individual contributions to the vertical motions by a number of forcing functions, in addition to the two (of the quasi-geostrophic theory) on the right hand side, are determined in a unique manner. The information gained by this procedure does yield considerable insight into the role of the individual mechanisms.

It might be asked why not portray fields of the forcing functions F_i rather than the partitioned vertical motion? The forcing functions by themselves are very cellular. The ω-field on the other hand is better defined. This is analogous, for instance, to a relation between $\nabla^2\psi$ and ψ. The former (vorticity) is more cellular while the ψ field may exhibit long waves. The forcing functions are proportional to $\nabla^2\sigma\omega$.

9. COMPUTATIONAL DETAILS

GRID POINTS

The five level model has a vertical staggering of variables. V, ψ, χ, ϕ, z, $\partial\psi/\partial t$, and $\partial\omega/\partial p$ appear at the 1000-, 800-, 600-, 400-, and 200-mb. surfaces. ω, T, θ, q, q_s and the forcing functions of the ω-equation appear at the 900-, 700-, 500-, and 300-mb. surfaces. In the horizontal, along the zonal direction there are 33 points of which the first 27 contain real initial data from analyzed weather maps and the last six provide a cyclic continuity such that points along 1 and 33 have identical values for dependent variables. Points 28, 29, 30, 31, and 32 are interpolated through use of information at points 1, 2, 26, and 27. In the meridional direction there are 15 points; points 1 and 15 are zonal walls at two latitude circles, where boundary conditions are specified. There is no staggering of variables in the horizontal. Each dependent variable is thus described over either 2,475 or 1,980 grid points.

BOUNDARY CONDITIONS

ψ, $\partial\psi/\partial t$, and χ are obtained from solutions of two dimensional Poisson type equations. Boundary conditions are applied at the meridional limits $y=y_1$ and $y=y_2$. These equations are independently solved for each vertical level. $\psi=gz/f$ is assumed at y_1 and y_2 if the balance equation is solved for observed geopotential distribution. If the observed wind field is used then the procedure is the following. A pseudo velocity potential χ is first obtained by solving,

$$\nabla^2\chi=-\left(\frac{\partial u^o}{\partial x}+\frac{\partial v^o}{\partial y}\right) \tag{68}$$

χ is assumed zero at y_1 and y_2, u^o and v^o are the components of the observed wind. Net outflow from the domain

$$\frac{\partial\bar{\chi}}{\partial n}=\frac{\oint\frac{\partial\chi}{\partial n}ds}{\oint ds} \tag{69}$$

is evaluated at y_1 and y_2. This is used to prescribe a boundary condition on ψ by letting,

$$\frac{\partial\psi}{\partial s}=-\frac{\partial\chi}{\partial n}+\frac{\partial\bar{\chi}}{\partial n} \tag{70}$$

and the relation

$$\psi=\psi_0+\int_0^s\frac{\partial\chi}{\partial s}ds$$

prescribes values at the boundaries $y=y_1$ and y_2. This assures no net outflow from the boundaries. This pseudo field of χ is discarded in the interior because of the unreliability of its divergence field (Hawkins and Rosenthal [4]).

For the solutions of $\partial\psi/\partial t$ and χ of the balance model we assume $\partial\psi/\partial t=\chi=0$ at $y=y_1$ and y_2, vertical motion ω is assumed$=0$ at $p=100$ mb. and $=0$ or $-g\rho$ $(\mathbf{V}\cdot\nabla h)$ at $p=1000$ mb.

RELAXATION PROCEDURE

The equations of the form

$$\nabla^2PQ+R\frac{\partial^2Q}{\partial p^2}=T$$

are assumed to be elliptic here. Let the finite difference analog of this equation be written by the equation,

$$\text{Residue}=\nabla^2PQ+R\frac{D^2Q}{Dp^2}-T \tag{71}$$

where $\nabla^2 PQ$ and D^2Q/Dp^2 are the finite difference analogs of ∇^2PQ and $\partial^2Q/\partial p^2$ respectively. In order to use the same general form of the Liebmann forward extrapolation technique, equation (71) may be rewritten in the form,

$$\text{Normalized Residue}=\left(\nabla^2PQ+R\frac{D^2Q}{Dp^2}-T\right)\frac{1}{(P|\nabla^2|)}. \tag{72}$$

The normalizing factor is the inverse of the coefficient of Q where $(|\nabla^2|)$ is the magnitude of the finite difference del square operator. It is a function of grid lengths and the map projection scale factors.

In the iterative procedure of the Liebmann relaxation, during each scan Q is modified by the relation

$$Q=Q+\alpha\cdot\text{Normalized Residue} \tag{73}$$

where α is the over-relaxation coefficient.

This procedure converges very rapidly provided a trial search is made to determine the optimum magnitude of α. No textbook rules determine the values of α in a general problem. α is found to vary for different grid distances, static stabilities, and ranges of Coriolis parameters. We have used

$$\psi\rightarrow\alpha=0.31$$

$$\omega\rightarrow\alpha=0.37$$

$$\chi\rightarrow\alpha=0.47$$

$$\frac{\partial\psi}{\partial t}\rightarrow\alpha=0.47$$

for the middle latitude winter storm investigations. In the Tropics values differ somewhat on the lower side.

MAP SCALE FACTORS

A map scale factor, M, appropriate for either a mercator or a Lambert conformal equation, is used in the tangent plane equations and the finite difference analogs.

TRUNCATION ERROR

Truncation errors of forcing functions of the various differential equations are different, but in each equation all the terms have the same maximum order of truncation error. For example, in the ω-equation the forcing function $\partial J(\psi,\ \zeta_a)/\partial p$ for prescribed ψ and ζ_a, has an error of the order of $\Delta x\Delta y\Delta p$.

Note also that

$$-2\frac{\partial}{\partial t}\frac{\partial}{\partial p}J(u_\psi,v_\psi)=-2\frac{\partial}{\partial p}\left[J\left(u_x,\frac{\partial v_\psi}{\partial t}\right)-J\left(v_\psi,\frac{\partial}{\partial t}u_\psi\right)\right]$$

also has the same order of error for prescribed u_ψ, v_ψ, and $\partial u_\psi/\partial t$, $\partial v_\psi/\partial t$. u_ψ and v_ψ are evaluated from the stream function ψ, and $\partial u_\psi/\partial t$ and $\partial v_\psi/\partial t$ are evaluated from the solution for $\partial\psi/\partial t$.

10. SOME CONCLUDING REMARKS

The preceding outline of a general balance model is used for diagnostic studies of several weather systems in high and low latitudes. See Krishnamurti [6]. In this issue of the *Monthly Weather Review* two applications are presented, one in middle latitudes (Krishnamurti [7]) and one in low latitudes (Baumhefner [2]).

Further studies of three dimensional motion field in the vicinity of the Equator where the typical Rossby number is of the order of unity can be carried out by using the information obtained from such a balance model to define the initial state for a primitive equation prediction model. Short range prediction yields useful information for $R_o>1$. We will present the results of such experiments in the near future.

The formulation of heating, friction, and terrain effects requires much further work; the present approach is very simple. The application we present here utilizes a horizontal mesh size of the order of 200 km.; small-scale processes in the vertical may be parameterized as indicated; there are several smaller scale processes in the horizontal, especially in middle latitudes, that are neither resolved by this mesh nor are parameterized in this study. Examples of such processes are usually found in the vicinity of intense jet streams where on a smaller scale maximum values of various terms of the dynamical equations may be present. Parameterization of such processes will be needed for improvement of short range synoptic scale forecasts.

ACKNOWLEDGMENTS

Principal financial support for the work described in this paper was obtained from Air Force Cambridge Research Laboratory, Bedford, Mass., during the period November 1964–November 1966. Dr. Carl Krietzberg of AFCRL and Professor Edwin Danielsen of Pennsylvania State University provided considerable encouragement for the continuation of this work.

Partial financial support was also received from ESSA grant number: E 65-67 (G) through Dr. Fred Shuman, National Meteorological Center, and from NASA grant number: NsG 237-62 through Professor Gordon MacDonald, Institute of Geophysics at UCLA.

Mr. John M. Brown, now a graduate student at MIT, was a part of many of our discussions. Mr. Robert M. Coie, a senior programer for IBM at Douglas Aircraft Company at Long Beach, Calif., was a consultant on some of the coding aspects. He provided considerable clarification and efficiency.

Computations were performed on the IBM 7094 computer; we are grateful to the Computing Facility of UCLA for their generous support.

REFERENCES

1. L. Berkofsky and E. A. Bertoni, "Mean Topographic Charts for the Entire Earth," *Bulletin of the American Meteorological Society*, vol. 36, No. 7, Sept. 1955, pp. 350–354.
2. D. Baumhefner, "Application of a Diagnostic Numerical Model to the Tropical Atmosphere," *Monthly Weather Review*, vol. 96, No. 4, Apr. 1968, pp. 218–228.
3. J. Charney and A. Eliassen, "On the Growth of the Hurricane Depression," *Journal of the Atmospheric Sciences*, vol. 21, No. 1, Jan. 1964, pp. 68–75.
4. H. F. Hawkins and S. L. Rosenthal, "On the Computation of Stream Functions From the Wind Field," *Monthly Weather Review*, vol. 93, No. 4, Apr. 1965, pp. 245–252.
5. W. C. Jacobs, "Large Scale Aspects of Energy Transformation Over the Oceans," *Compendium of Meteorology*, American Meteorological Society, Boston, 1951, pp. 1057–1070.
6. T. N. Krishnamurti (Principle Investigator), *Diagnostic Studies of Weather Systems of Low and High Latitudes (Rossby Number <1)*, Final Report, Contract AF19(628)-4777, AFCRL-67-0128, Dept. of Meteorology, University of California at Los Angeles, Apr. 1967, 360 pp.
7. T. N. Krishnamurti, "A Study of a Developing Wave Cyclone," *Monthly Weather Review*, vol. 96, No. 4, Apr. 1968, pp. 208–217.
8. T. N. Krishnamurti, "A Calculation of Percentage Area Covered by Convective Clouds," *Journal of Applied Meteorology*, vol. 7 (in press).
9. H. L. Kuo, "On Formation and Intensification of Tropical Cyclones Through Latent Heat Release by Cumulus Convection," *Journal of the Atmospheric Sciences*, vol. 22, No. 1, Jan. 1965, pp. 40–63.
10. E. N. Lorenz, "Energy and Numerical Weather Prediction," *Tellus*, vol. 12, No. 4, Nov. 1960, pp. 364–373.
11. J. Paegle, "Numerical Calculation of Three Dimensional Trajectories Utilizing the Balanced Horizontal and Vertical Motions," *Diagnostic Studies of Weather Systems at Low and High Latitudes (Rossby Number <1)*, Final Report, Contract AF19(628)-4777, AFCRL-67-0128, Dept. of Meteorology, University of California at Los Angeles, Apr. 1967, pp. 1–21.
12. S. Petterssen, *Weather Analysis and Forecasting*, McGraw-Hill Book Co., Inc., New York, 2d Edition, 1956, 428 pp. (see Chapter 16, pp. 320–338).
13. N. A. Phillips, "Geostrophic Motion," *Reviews of Geophysics*, Washington, D.C., vol. 1, No. 2, May 1963, pp. 123–176.
14. F. G. Shuman, "Numerical Methods in Weather Prediction, The Balance Equation," *Monthly Weather Review*, vol. 85 No. 10, Oct. 1957, pp. 329–332.
15. R. C. Sutcliffe, "A Contribution to the Problem of Development," *Quarterly Journal of the Royal Meteorological Society*, vol. 73, No. 317/318, July/Oct. 1947, pp. 370–383.

[Received May 31, 1967; revised December 20, 1967]